Basic (Low-Level) DIP
-- Part I
Image Enhancement in the Spatial Domain

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-- REU Site Program in CVMA
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Outline

- Point Processing
  - Power-law transform
  - Piecewise linear transform
  - Histogram equalization
- Some important statistical concepts
- Mask Processing
  - Filtering and Convolution
  - Smoothing Spatial Filtering (Averaging Filtering)
  - Order Statistics Filtering (Median Filtering)
  - Sharpening Spatial Filtering

Introduction

- Objective:

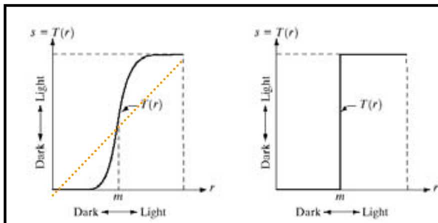
  To process an image so that the result is more suitable than the original image for a **specific** application. → **Problem Oriented**
  - To suppress undesired distortions
  - Emphasize and sharpen image features for display and analysis
- There is no general theory of image enhancement. When an image is processed for visual interpretation, the viewer is the ultimate judge of how well a particular method works. → **Subjective Process**

- Two broad categories of image enhancement approaches
  - **Spatial Domain Methods**: Direct manipulate pixels in an image.
  - **Frequency Domain Methods**: Modify the Fourier or Wavelet transform of an image.
- Normally, enhancement techniques use various combinations of methods from these two categories.

Spatial Domain Method:
Point Processing

- Point processing modifies each pixel intensity independently based on a function *T*. Let the intensity of pixel before modification be *r* and after modification be *s*, then $s = T(r)$.
- *T* is often referred to as:

  Gray-level (Intensity or Mapping) transformation function.



a b
FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

(a) Contrast Stretching (b)Thresholding

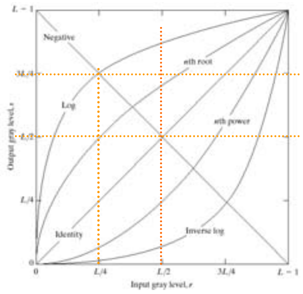
What is the general effect after applying the (a) function?

Properties of *T*:
- **Single value and monotonically** increasing (or decreasing in the case to get the negative) in the interval $0 <= r <= L-1$
- $0<=T(r) <= L-1$ for $0 <= r <= L-1$

Point Processing Methods
-- Basic Gray Level Transformations

Basic Gray-level Transformation Function Used for Image Enhancement

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.



Point Processing Methods
-- Basic Gray Level Transformations (Cont.)

1) Image Negative: $s = L - 1 - r$;

2) Log Transformations: $s = c \log(1 + r)$;
where c is a constant.

3) Power-Law Transformation

$s = cr^{\gamma}$ where c and γ are positive constants.

4) Piecewise-Linear Transformation

The goal is to increase the dynamic range.

$$s = \begin{cases} \alpha r & \text{for } 0 <= r < r1 \\ \beta(r - r1) + s1 & \text{for } r1 <= r < r2 \\ \gamma(r - r2) + s2 & \text{for } r2 <= r < L \end{cases}$$

Point Processing Methods
-- Power-Law Transformation

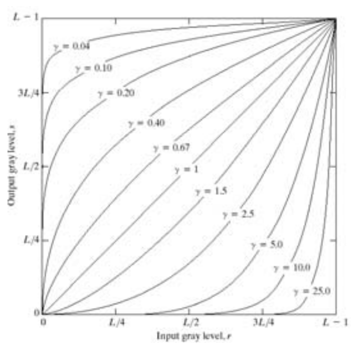
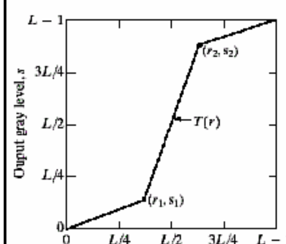


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ (c = 1 in all cases).

The curves generated with values of γ > 1 have exactly the opposite effect as those generated with values of γ < 1 .

Point Processing Methods
-- Piecewise Linear Transformation



The goal is to increase the dynamic range.

$$s = \begin{cases} \alpha r & \text{for } 0 <= r < r1 \\ \beta(r - r1) + s1 & \text{for } r1 <= r < r2 \\ \gamma(r - r2) + s2 & \text{for } r2 <= r < L \end{cases}$$

1. How about r1 = s1 and r2 = s2?
2. How about r1 = r2, s1 = 0, and s2 = L-1?
3. How about r1 = Minimum gray level, s1 = 0 and r2 = maximum gray level, s2 = L-1?

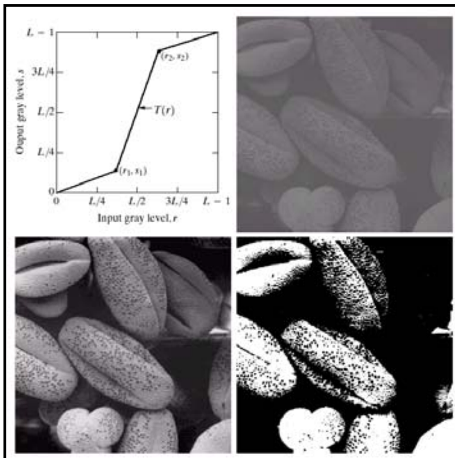
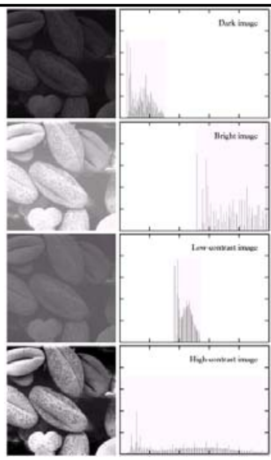


FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Point Processing Methods
-- Histogram Processing

- The **histogram** of a digital image with gray levels in the range [0, L-1] is a discrete function $h(r_k) = n_k$, where r_k is the kth gray level and n_k is the number of pixels in the image having gray level r_k.
- A **normalized histogram** is given by $p(r_k) = n_k/n$ for k = 0, 1, …, L-1 and n is the total number of pixels in the image. That is, $p(r_k)$ gives an estimate of the probability of occurrence of gray level r_k.



It is reasonable to conclude that an image whose pixels tend to **occupy the entire range** of possible gray levels and, in addition, tend to be **distributed uniformly**, will have an appearance of high contrast and will exhibit a large variety of gray tones.

The net effect will be an image that shows a great deal of gray-level detail and has a high dynamic range.

Point Processing Methods
-- Properties of the Image Histogram

- Histogram clustered at the low end: Dark Image
- Histogram clustered at the high end: Bright Image
- Histogram with a small spread: Low contrast Image
- Histogram with a wide spread: High contrast Image

Point Processing Methods
-- Histogram Equalization

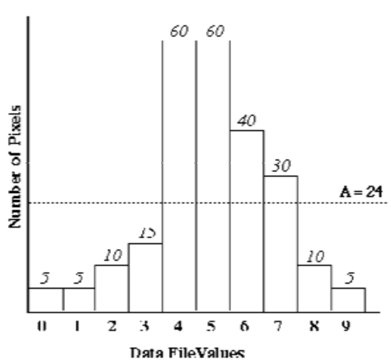
- Histogram equalization is to increase the dynamic range of an image by an **non-linear** intensity transformation function such that the histogram of the transformed image covers the whole dynamic range with equal probability.
  - ➔ Stretch the contrast by redistributing the gray-level values uniformly.
  - ➔ It is fully automatic compared to other contrast stretching techniques.

Point Processing Methods
-- Discrete Histogram Equalization

Given an image of M x N with maximum gray level being L–1:

1. Obtain the histogram H(k), k = 0, 1, …, L-1
2. Compute the cumulative normalized histogram T(k): $T(k) = \sum_{i=0}^{k} H(i) / MN$
3. Compute the transformed intensity: $g_k = (L-1) * T(k)$.
4. Scan the image and set the pixel with the intensity k to g_k.

Before Equalization



Total Number of Pixels:
5+5+10+15+60+60+40+30+10+5 = 240.

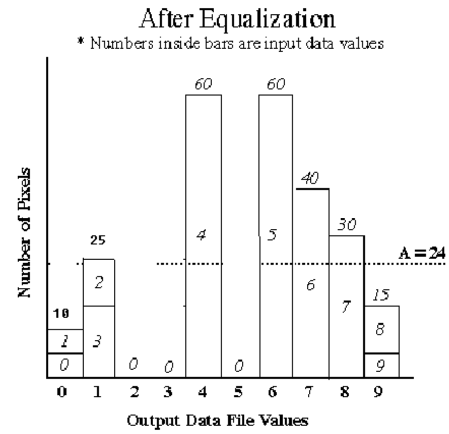
Step 1: H(0) = 5 ; H(1) = 5 ; H(2) = 10 ;
H(3) = 15 ; H(4) = 60 ; H(5) = 60 ;
H(6) = 40 ; H(7) = 30 ; H(8) = 10 ;
H(9) = 5 ;

Step 2: T(0) = 5/240 ; T(1) = 10/240 ;
T(2) = 20/240 ; T(3) = 35/240 ;
T(4) = 95/240 ; T(5) = 155/240 ;
T(6) = 195/240 ; T(7) = 225/240 ;
T(8) = 235/240 ; T(9) = 240/240 ;

- Step 3: G0 = 0.1875 →0; G1 = 0.3750→0;
  G2 = 0.7500→1 ; G3 = 1.3125→1;
  G4 = 3.5625→4 ; G5 = 5.8125→6;
  G6 = 7.3125→7 ; G7 = 8.4375→8 ;
  G8 = 8.8125→9 ; G9 = 9.0000→9 ;
- Step 4:

Original Intensity →New Intensity → Number

0→0→5; 1→0→5 ; 2→1→10 ;
3→1→15; 4→4→60; 5→6→60 ;
6→7→40; 7→8→30; 8→9→10;
9→9→5;



Several Important Statistical Concepts

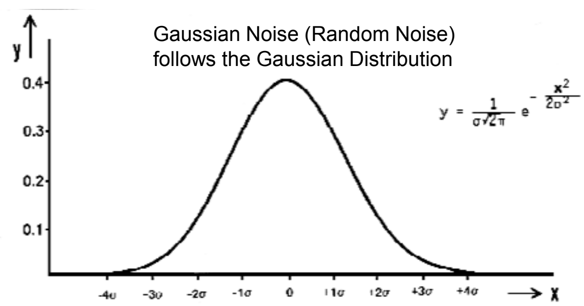
Mean (Average): $m = \sum_{i=0}^{L-1} r_i \, p(r_i);$

The nth moment of r about its mean: $\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n \, p(r_i);$

Variance $\mu_2(r) = \sigma^2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 \, p(r_i).$

In a digital image, the mean is a measure of average gray level; the variance (or standard deviation, which is a square root of the variance) is a measure of average contrast

Gaussian Noise (Random Noise) follows the Gaussian Distribution

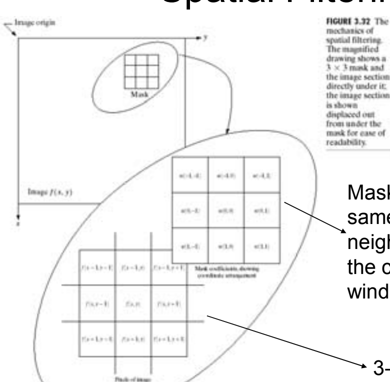


The Gaussian distribution shows the probability *y* of finding a deviation *x* from the mean (*x* = 0), according to the equation stated, where *e* is the base of natural logarithms, and *σ* is the standard deviation. The probability of larger deviations can be seen to decrease rapidly.

Mask Processing (Filtering) Methods

- *T* operates on a neighborhood of pixels.
- Neighborhood about a point *(x,y)* normally is a square or rectangular subimage area centered at *(x, y).*
- The general approach is to use a function of the values of *f* in a predefined neighborhood of *(x, y)* to determine the value of *g* at *(x, y).*

Spatial Filtering



Mask (filter) with the same dimensions as the neighborhood contains the coefficients (i.e., window)

3-by-3 window of the pixel f(x, y)

- Spatial filtering alters image intensity of a pixel based on its intensity, the intensities of the neighboring pixels, and the coefficients of the corresponding mask. That is:
  - For a linear spatial filtering, the response is given by a sum of products of the filter coefficients with the corresponding image pixels in the area spanned by the filter mask.
  - In general, linear filtering of an image f of size MxN with a filter mask of size mxn is given by the expression:

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)$$

$$\text{where } a=(m-1)/2 \text{ and } b=(n-1)/2$$

  - It is accomplished by convolving the mask with the original image.

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Original Image:

4	6	8	9	10
1	3	5	6	9
2	4	6	12	12
4	8	10	14	6

1. What happens when the center of the filter approaches the border of the image?
2. How to handle this situation?

Mask 1:

1	1	1
1	1	1
1	1	1

Mask 2:

1	2	3
1	4	1
1	3	2

Convolution Result:

14	27	37	47	34
20	39	59	77	58
22	43	68	80	59
18	34	54	60	44

Convolution Result:

31	56	77	95	82
47	84	124	160	119
51	94	137	174	116
40	74	114	138	74

Matlab Function filter2 can do this convolution!

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Three types of Spatial Filters:

- Low-pass filter: Enhance low frequency components of the image while eliminating or reducing high frequency components.
- High-pass filter: Enhance high frequency components of the image while eliminating or reducing low frequency components.
- Band-pass filter: Enhance certain range of frequency components of the image while eliminating or reducing frequency components beyond the range.

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Smoothing Spatial Filtering

- It is a linear low-pass filter
- A standard averaging filter replaces the value of every pixel in an image by the average of the gray levels in the window defined by the filter mask. This process results in an image with reduced “sharp” transitions in gray levels.
- A weighted averaging filter uses different coefficients at different spatial locations.

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$\frac{1}{9}\times$

1	1	1
1	1	1
1	1	1

$\frac{1}{16}\times$

1	2	1
2	4	2
1	2	1

Standard vs. Weighted Averaging Filter

1. What is the sum of all the coefficients in the mask?
2. What is the characteristic of the weighted averaging filter in terms of the coefficient values?
3. What conclusion can you draw on the smoothing spatial filters?

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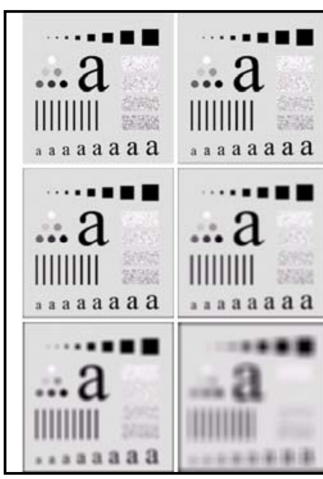

Averaging filter results with the mask size of 3x3, 5x5, 9x9, 15x15, and 35x35.

1. What is the desirable feature of the averaging filter?
2. What is the undesirable side effect of the averaging filter?
3. What is the effect of the size of the filter?
4. How to determine the best filter size for a specific image?

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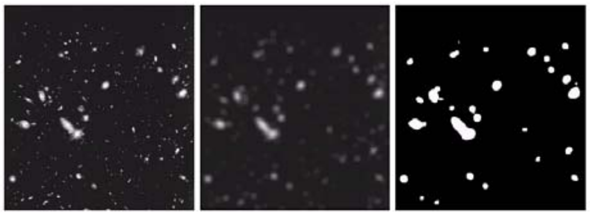
Ex1: Find the larger and brighter objects in the image



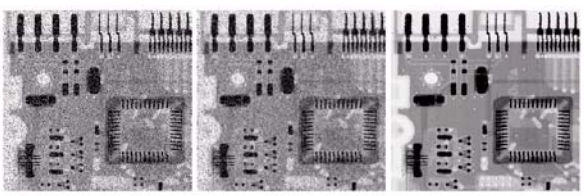
a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

1. Image is processed by a 15-by-15 averaging mask.
2. Thresholding with a threshold value equal to 25% of the highest intensity in the blurred image

Order Statistics Filtering

- It is a nonlinear low-pass filter
- Its filtering result is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.
  – Median filter: Replace center pixel with median of the gray levels in the window of that pixel (the original pixel value is included in the computation of the median).



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

1. What is the difference between the averaging filter and median filter results?
2. Which filter has more computational cost?
3. Which filter will you choose to remove the additive salt-and-pepper noise?

Order Statistics Filtering: Illustration

The filter can be shown as a convolution mask, for example, for an average 3x3 filter

What does this filter look like?

image	averaging filter result	median filter result
9 9 9 0 0 0	. . . . . .	. . . . . .
9 9 9 0 0 0	. 8 5 3 0 .	. 9 9 0 0 .
9 0 9 0 0 0	. 8 5 4 1 .	. 9 9 0 0 .
9 9 9 0 9 0	. 8 5 4 1 .	. 9 9 0 0 .
9 9 9 0 0 0	. 9 6 4 1 .	. 9 9 0 0 .
9 9 9 0 0 0	. . . . . .	. . . . . .

- **Determine the size of the mask**
  1. The size of the mask must be larger than the scale of the noise but smaller than the dimensions of any structure in the image that is important to subsequent analysis. That is, features (e.g. lines or spots) that are smaller than half the mask can be selectively eliminated as noise (or at least not features of interest).
  2. The larger the mask, the longer the ranking process (for the median filter) or the computational process (for the averaging filter) takes
- **Comparison between averaging and median filters:**

The median filter is superior to the smoothing filter in that it does not smooth or blur the boundaries of regions or features in the image.

Sharpening Spatial Filtering

- The goal of the sharpening is to highlight fine detail in an image or to enhance detail that has been blurred.
  – Sharpening could be accomplished by spatial differentiation.
  – Image differentiation enhances edges and other discontinuities (such as noise) and deemphasizes areas with slowly varying gray-level values.

• The Laplacian Filter

1. What is the sum of the entries in the mask?
2. What is the difference between the sum of the smoothing and sharpening spatial filters?

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1
0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

a b
c d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

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• The basic way in which we use the Laplacian for image enhancement is as follows:

$$g(x,y)=\begin{cases} f(x,y)-\nabla^2 f(x,y) & \text{if the center coefficient of Laplacian mask is negative} \\ f(x,y)+\nabla^2 f(x,y) & \text{if the center coefficient of Laplacian mask is positive} \end{cases}$$

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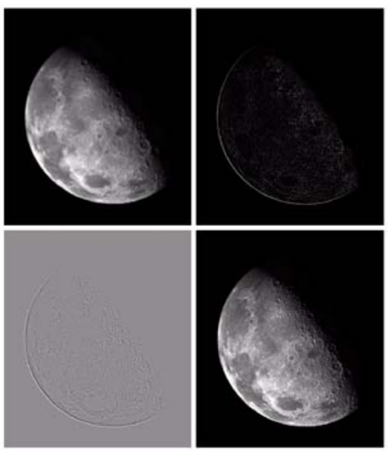

a b
c d

FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)

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a
b c
d e

FIGURE 3.44 A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Roberts Cross-Gradient Operators

−1	0	0	−1
0	1	1	0

Gx=Z9 – Z5 ;
Gy = Z8 – Z6 ;

Sobel Operators

−1	−2	−1	−1	0	1
0	0	0	−2	0	2
1	2	1	−1	0	1

Gx=(z7+2Z8+z9)-(z1+2Z2+Z3) ;

Gy=(Z3+2Z6+Z9)-(Z1+2Z4+Z7) ;

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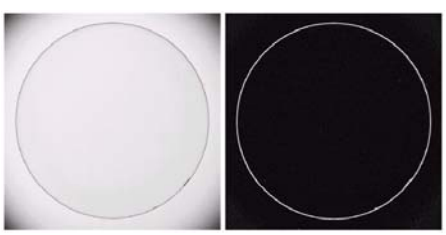

a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

The edge defects are also quite visible, but with the added advantage that constant or slowly varying shades of gray have been eliminated.

The ability to enhance small discontinuities in an otherwise flat gray field is another important feature of the gradient

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Useful Matlab Commands

- imadjust
- imhist, histeq, hist
- mean, var, median, max, min, std
- fspecial,imfilter, filter2
- edge

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